Kuwait University | Math 102 | July 17, 2006
Dept of Math & Comp Sci | Second Examination | Duration: 90 minutes

The use of calculator is not allowed. Turn off your mobile phone or pager

1. Evaluate the following integrals. **(3 points each)**

(a) $\displaystyle\int \frac{x^5}{\sqrt{1-x^3}}\,dx$

(b) $\displaystyle\int \frac{\sqrt{x^2-2x-8}}{x-1}\,dx$

(c) $\displaystyle\int (\csc^3 2x)\sin^6 x\,dx$

(d) $\displaystyle\int \frac{x^2-x+2}{x^3+2x^2+2x}\,dx$

(e) $\displaystyle\int \frac{2+\tan\frac{x}{2}}{2\sin x+2\cos x+3}\,dx$

(f) $\displaystyle\int \frac{2x}{1+\sqrt[3]{3x^2-1}}\,dx$

2. Determine if the improper integral $\displaystyle\int_{-\infty}^{0} \frac{e^x}{\sqrt{1+e^{-2x}}}\,dx$ is convergent or divergent, and find its value if it is convergent. **(3 points)**

3. Find the length of the arc of $f(x)=\ln(\sec x)$, $0 \le x \le \frac{\pi}{4}$. **(2 points)**

4. Find the surface area of the solid obtained by revolving the curve $f(x)=\cosh x$, $0 \le x \le 1$ about the y-axis.

(2 points)

f) let $3x^2-1 = u^3 \Rightarrow 6x\,dx = 3u^2du \Rightarrow 2x\,dx = u^2du$.

$\int \frac{u^2du}{1+u} = \int \left(u-1+\frac{1}{u+1}\right)du = \frac{u^2}{2} - u + \ln|u+1| +$

$= \frac{1}{2}(3x^2-1)^{2/3} - (3x^2-1)^{\frac{1}{3}} + \ln\left|(3x^2-1)^{\frac{1}{3}}+1\right| + C$

Q2 $\int_{-\infty}^{0} \frac{e^{2x}}{\sqrt{e^{2x}+1}} = \lim_{t\to-\infty} \int_t^0 \frac{e^{2x}}{\sqrt{e^{2x}+1}}dx$

$= \lim_{t\to-\infty} \left|\sqrt{e^{2x}+1}\right|_t^0 = \lim_{t\to-\infty}\left[\sqrt{2} - \sqrt{e^{2t}+1}\right]$

$= \sqrt{2}-1$ conv.

Q3 $f'(x) = \tan x$, $1+f'^2(x) = 1+\tan^2 x = \sec^2 x$

$L = \int_0^{\pi/4} \sec x\,dx = \left|\ln|\sec x+\tan x|\right|_0^{\pi/4}$

$= \ln(\sqrt{2}+1)$ Ans

Q4 $f'(x) = \sinh x$ $1+f'^2(x) = 1+\sinh^2 x = \cosh^2 x$.

$S = 2\pi\int_0^1 x\cosh x\,dx = 2\pi\left|x\sinh x - \cosh x\right|_0^1$

$= 2\pi\left[(\sinh 1-\cosh 1) - (0-\cosh 0)\right]$

$= 2\pi\left[\frac{e^1-e^{-1}}{2} - \frac{e^1+e^{-1}}{2} + 1\right]$

$= 2\pi\left[-\frac{2e^{-1}}{2}+1\right] = 2\pi(1-e^{-1})$ Ans

Taslim

Q1 (a) $\int \frac{x^5}{\sqrt{1-x^3}}dx$ let $1-x^3 = u^2 \Rightarrow -3x^2dx = 2u\,du$

$x^2dx = -\frac{2}{3}u\,du$

$\int \frac{x^3 \cdot x^2dx}{\sqrt{1-x^3}} = \int \frac{1-u^2}{u}\cdot -\frac{2}{3}u\,du = -\frac{2}{3}\left[u-\frac{u^3}{3}\right]+C = -\frac{2}{3}\left[\sqrt{1-x^3} - \frac{1}{3}(1-x^3)\right.$

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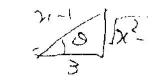
b) $\int \frac{\sqrt{x^2-2x-8}}{x-1}dx = \int \frac{\sqrt{(x-1)^2-3^2}}{x-1}dx$ Put $x-1 = 3\sec\theta$

$dx = 3\sec\theta\tan\theta$

$= \int \frac{3\tan\theta}{3\sec\theta}3\sec\theta\tan\theta\,d\theta = 3\int\tan^2\theta\,d\theta = 3\int(\sec^2\theta-1)\,d\theta$

$= 3\tan\theta - 3\theta + C$

$= 3\frac{\sqrt{x^2-2x-8}}{3} - 3\sec^{-1}\frac{x-1}{3} + C$



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c) $\int \csc^3 x \sin^6 x\,dx = \frac{1}{8}\int \frac{1}{\sin^3 x\cos^3 x}\cdot\sin^6 x\,dx = \frac{1}{8}\int\frac{\sin^3 x}{\cos^3 x}dx$

$= \frac{1}{8}\int\frac{1-\cos^2 x}{\cos^3 x}\cdot\sin x\,dx$ let $\cos x = u \Rightarrow -\sin x\,dx = du$

$-\frac{1}{8}\int\frac{1-u^2}{u^3}du = -\frac{1}{8}\int(u^{-3}-\frac{1}{u})du = -\frac{1}{8}\left[\frac{u^{-2}}{-2} - \ln u\right]+$

$= -\frac{1}{8}\left[\frac{-1}{2u^2} - \ln|u|\right]+C$

$= -\frac{1}{8}\left[\frac{-1}{2\cos^2 x} - \ln\cos x\right]+C = -\frac{1}{8}\left(-\frac{1}{2}\sec^2 x + \ln\sec x\right)$

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d) $\int\frac{x^2-x+2}{x(x^2+2x+2)}dx = \int\frac{1}{x}dx - 3\int\frac{1}{x^2+2x+2}dx$

$= \int\frac{1}{x}dx - 3\int\frac{1}{(x+1)^2+1^2}dx = \ln x - 3\tan^{-1}(x+1)+C$ Ans

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e) $\int\frac{2+u}{\left(\frac{4u}{1+u^2}+\frac{2-2u^2}{1+u^2}+3\right)}\cdot\frac{2du}{1+u^2} = \int\frac{2u+4}{4u+2-2u^2+3+3u^2}du$

$= \int\frac{2u+4}{u^2+4u+5} = \ln(u^2+4u+5)+C$

$= \ln(\tan^2\frac{x}{2}+4\tan\frac{x}{2}+5)+C$ Ans

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